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Relevant Deontic Logic Reconsidered

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Abstract
In this paper, we explore relevant deontic logics with a primitive obligation operator. We critically assess Goble's proposals for relevant deontic logics and propose alternative minimal deontic systems. We then examine the status of the ought implies can principle. Finally, we investigate options for strong deontic logics.

1 Introduction

Deontic logics are logics in which normative concepts are represented by modal operators. A paradigm example is the logic of obligation and permission, where one adds an operator $\mathcal{O}$ to the language to stand for 'it is obligatory that'. One then defines further concepts in terms of that, such as permission ($\neg\mathcal{O}\neg A$) and prohibition ($\mathcal{O}\neg A$). There are further normative concepts that one can represent through the addition of more operators, such as the binary conditional obligation operator, $\mathcal{O}(\cdot|\cdot)$, although we will not do so here.[1]

Deontic logics are typically studied over classical logic as the base logic. This is not, however, required, and investigation of deontic logics over non-classical base logics is a promising idea to explore. While in the classical context, there is a plausible basic deontic logic, when one moves away from the classical context, there is a question of which principles should be adopted for such a basic deontic logic.[2] In this paper, we will identify a plausible minimal deontic relevant logic, and then we will discuss some ways to strengthen it.

In this paper, we will start by supplying some background on deontic logics (§2). Then, in §3 we will go over the basics of relevant logics. Following this, in §4, we turn to the combination of relevant and deontic logics, discussing Goble's proposals for relevant deontic logics and offering our own proposal for minimal relevant deontic logics. In §5, we examine Routley and Plumwood's discussion of the principle that

[1] Goble (1999) explores adding a conditional obligation operator to relevant logics, but we will not follow suit here.

[2] Throughout this paper, 'principle' will be used as a neutral term for both axioms and rules.

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ought implies can, what they call "Kant's law."[3] We argue that their concerns about Kant's law are misguided in the context of relevant logics. After this, in §6, we look at some ways of strengthening the basic relevant deontic logic. Finally, we conclude in §7 with a brief summary and suggestions for future work.

2 Deontic Logics

Throughout this paper, we work in a propositional language with a countable set of atoms and the logical vocabulary $\{\neg, \wedge, \vee, \to, \mathcal{O}\}$. In §5, we further extend the language with a singulary possibility operator, $\Diamond$. When presenting rules for a logic, we will use '$\Rightarrow$', with the premises to the left of the arrow and the conclusion to the right.

Deontic logics have mostly been studied using classical logic as the base logic. In the context of classical logic, it is common to use Kripke frames with an accessibility relation to model deontic logics.

Definition 2.1 A *deontic Kripke frame* is $\langle W, S\rangle$ where $W \neq \emptyset$ and S is a binary relation that is serial, i.e. satisfying

- for all $w \in W$, there is $x \in W$ such that Swx.

A *deontic Kripke model* is a frame equipped with a valuation V that maps atoms to subsets of W.

The valuations are extended to the whole language inductively as follows.

- $w \Vdash p$ iff $w \in V(p)$
- $w \Vdash \neg B$ iff $w \nVdash B$
- $w \Vdash B \wedge C$ iff $w \Vdash B$ and $w \Vdash C$
- $w \Vdash B \vee C$ iff $w \Vdash B$ or $w \Vdash C$
- $w \Vdash B \to C$ iff $w \nVdash B$ or $w \Vdash C$
- $w \Vdash \mathcal{O}B$ iff for all $x \in W$, if Swx, then $x \Vdash B$

Validity for these models is defined as truth at all worlds in all models.[4]

The logic that results from these frames is the basic deontic logic, KD, which adds to K the axiom (D) $\mathcal{O}\neg A \to \neg\mathcal{O}A$.

- $\mathcal{O}(A \to B) \to (\mathcal{O}A \to \mathcal{O}B)$ (K)
- $(\mathcal{O}A \wedge \mathcal{O}B) \to \mathcal{O}(A \wedge B)$ (Agg)
- $\mathcal{O}A \to \neg\mathcal{O}\neg A$ (D)
- $A \to B \Rightarrow \mathcal{O}A \to \mathcal{O}B$ (Mono)
- $A \Rightarrow \mathcal{O}A$ (Nec)

[3] A note about the names is in order. While Richard Routley changed his name to Richard Sylvan, shortly after Valerie Routley changed her name to Valerie Plumwood, the article we discuss by them was published with "Routley and Plumwood" as the authors. As this is the only article by them we discuss at length, we will refer to them by the published names.

[4] Throughout this paper, logics are treated as sets of logical truths. This is the FMLA framework of Humberstone (2011, 103ff.). We will use 'theorem' and 'valid' for formulas in a logic, regardless of whether the logic was defined in terms of proofs or in terms of frames.

This list of principles is slightly redundant, but the redundancies are there to facilitate some later discussion and comparison. We will note that (D) has an equivalent formulation that is equivalent in the logics to be considered later, namely $\mathcal{O}\neg A \to \neg\mathcal{O}A$.[5] For purposes of later discussion, it is also worth noting that (D) has as a consequence $\neg(\mathcal{O}A \wedge \mathcal{O}\neg A)$.

The logic can be strengthened in various ways. One way to strengthen the logic is restrict to frames where S is

- serial ($\forall x \in W \exists y \in W$ such that Sxy),
- transitive ($\forall x, y, z \in W$, if Sxy and Syz, then Sxz), and
- Euclidean ($\forall x, y, z \in W$, if Sxy and Sxz, then Syz).

These frames will validate the logic KD45, which adds the axioms

- $\mathcal{O}A \to \mathcal{O}\mathcal{O}A$ (4) and
- $\neg\mathcal{O}A \to \mathcal{O}\neg\mathcal{O}A$ (5)

to KD. One can, alternatively, replace S with a designated non-empty set $X \subseteq W$, and change the verification condition for $\mathcal{O}$ to quantify over X. These are the so-called semi-simplified frames, and the verification condition appropriate to them is

- $w \Vdash \mathcal{O}B$ iff for all $x \in X$, $x \Vdash B$.

These frames also validate KD45. We will return to them in §6. Let us turn to the basics of relevant logics.

3 Relevant Logics

Relevant logics make up a family of non-classical logics.[6] The distinguishing feature of a relevant logic is that the implication connective requires a strong connection between antecedent and consequent. This is reflected in Belnap's variable-sharing property. Having Belnap's variable-sharing property is often taken as a necessary condition on being a relevant logic.[7]

Definition 3.1 A logic L *enjoys the variable-sharing property* iff for any formulas A, B, if $A \to B$ is a theorem of L, then A and B share a propositional variable

The variable-sharing property captures a sense of formal relevance. Some standard logics, such as classical logic and intuitionistic logic, do not enjoy the variable-sharing property.

The basic relevant logic B is axiomatized using the following axioms and rules.

(B1) $A \to A$
(B2) $A \wedge B \to A$, $A \wedge B \to B$

[5] There are other classically equivalent formulations that are not equivalent in logics considered later in the paper, such as $\neg\mathcal{O}\bot$ and $\neg\mathcal{O}(A \wedge \neg A)$.

[6] For overviews of relevant logic, see Read (1988), Dunn and Restall (2002), Mares (2004; 2020), Bimbó (2007), Logan (2024), or Standefer (2024; 2026b).

[7] Standefer (2025) discusses the variable-sharing property and proposes taking it as a sufficient condition on being a relevant logic as well.

(B3) $((A \to B) \wedge (A \to C)) \to (A \to (B \wedge C))$
(B4) $A \to A \vee B, B \to A \vee B$
(B5) $((A \to C) \wedge (B \to C)) \to ((A \vee B) \to C)$
(B6) $A \wedge (B \vee C) \to (A \wedge B) \vee (A \wedge C)$
(B7) $\neg\neg A \to A$
(B8) $A, A \to B \Rightarrow B$
(B9) $A, B \Rightarrow A \wedge B$
(B10) $A \to \neg B \Rightarrow B \to \neg A$
(B11) $A \to B \Rightarrow (C \to A) \to (C \to B)$
(B12) $A \to B \Rightarrow (B \to C) \to (A \to C)$

While there are many stronger relevant logics that are often considered, we will only need to discuss the logic R. The logic R is obtained by adding to B the following axioms.

(R1) $(A \to \neg B) \to (B \to \neg A)$
(R2) $(A \to (A \to B)) \to (A \to B)$
(R3) $A \to ((A \to B) \to B)$
(R4) $(A \to B) \to ((B \to C) \to (A \to C))$

An important fact about R is that it enjoys the variable-sharing property.[8] An important fact about variable sharing, which will come up below, is that it is preserved to sublogics. Therefore, B also enjoys the variable-sharing property.

The frames for relevant logics are slightly different from the Kripke frames for (classically-based) modal logics.

Definition 3.2 A *ternary relational frame* F is a quadruple $\langle K, N, R, ^* \rangle$, where $K \neq \emptyset$, $N \subseteq K$, $R \subseteq K^3$, $^* : K \mapsto K$, where $a \leq b$ is defined as $\exists x \in N\, Rxab$, and where

(B1) $\leq$ is a partial order,
(B2) $a^{**} = a$,
(B3) $a \leq b$ only if $b^* \leq a^*$, and
(B4) if $d \leq a$, $e \leq b$, $c \leq f$, and $Rabc$, then $Rdef$.

A ternary relational model is a frame F equipped with a valuation function V from atoms to subsets of K that has the feature that if $a \leq b$ and $a \in V(p)$, then $b \in V(p)$. Such model is said to be built on the frame F.

The valuation function is extended to a verification relation on the whole language as follows.

- $a \Vdash p$ iff $a \in V(p)$
- $a \Vdash \neg B$ iff $a^* \nVdash B$
- $a \Vdash B \wedge C$ iff $a \Vdash B$ and $a \Vdash C$
- $a \Vdash B \vee C$ iff $a \Vdash B$ or $a \Vdash C$
- $a \Vdash B \to C$ iff for all $b, c \in K$, if $Rabc$ and $b \Vdash B$, then $c \Vdash C$

One can get frames for R by adding the following conditions.

[8] See Anderson and Belnap (1975, 252-254) or Routley et al. (1982, 250). See also Brady (1984), Robles and Méndez (2011; 2012), Logan (2021; 2022), Ferguson and Logan (2025), and Standefer et al. (2025) for general discussion of variable sharing.

- For all $a, b, c \in K$, if $Rabc$, then Rac^*b^*.
- For all $a, b, c \in K$, if $Rabc$, then $\exists x \in K(Rabx \text{ and } Rxbc)$.
- For all $a, b, c \in K$, if $Rabc$, then $Rbac$.
- For all $a, b, c, d \in K$, if $\exists x \in K(Rabx \text{ and } Rxcd)$, then $\exists x \in K(Raxd \text{ and } Rbcx)$.

Soundness and completeness results for R can be obtained with respect to the frames obeying these conditions.

There are two important lemmas concerning the ternary relational models that we note here without proof.

Lemma 3.1 *For all formulas A, if $a \leq b$ and $a \Vdash A$, then $b \Vdash A$.*

Lemma 3.2 *The following are equivalent in ternary relational models.*

- *For all $a \in N$, $a \Vdash A \to B$.*
- *For all $a \in K$, if $a \Vdash A$, then $a \Vdash B$.*

These two lemmas carry over to all the extensions of ternary relational models considered below.

The concepts of counterexample, and so validity, are defined in terms of the set N, rather than the set K.

Definition 3.3 A model M is a counterexample to a formula A iff for some $a \in N$, $a \nVdash A$.

A formula A is valid in a class of frames $\mathcal{C}$ iff there is no model M built on a frame F in $\mathcal{C}$ such that M is a counterexample to A.

When A is valid in a class of frames $\mathcal{C}$, we write $\models_{\mathcal{C}} A$

The logic B is sound and complete with respect to the class of all ternary relational frames. Additionally, the logic R is sound and complete with respect to the class of ternary relational frames obeying the appropriate conditions.[9]

There is one additional concept that we will introduce before proceeding to the relevant deontic logics.

Definition 3.4 A set X of formulas is an L-theory iff (i) whenever $A \to B$ is a theorem of L and $A \in X$, then $B \in X$, and (ii) if $A \in X$ and $B \in X$, then $A \wedge B \in X$.

We will use L-theories, for various choices of L, in our discussion below to formalize claims about different deontic situations. With that background in place, we next consider adding deontic principles to our base relevant logics.

4 Relevant Deontic Logics

In this section, we will turn to deontic extensions of relevant logics. There are two main ways of proceeding to study relevant deontic logics. One is to add a normative constant, $\mathfrak{v}$, to the language, to stand for a violation, possibly along with a necessity operator $\Box$, and *define* $\mathcal{O}A$ as $\Box(\neg A \to \mathfrak{v})$. This is the Anderson-Kanger reduction, and it has

[9] For the conditions, see (Standefer, 2026b, ch. 3), among others.

Table 1 Goble's logics

Name	Deontic principle	OR.1	OR.2	DR.1	DR.2
(Agg)	$(\mathcal{O}A \wedge \mathcal{O}B) \to \mathcal{O}(A \wedge B)$	✓	✓	✓	✓
(K)	$\mathcal{O}(A \to B) \to (\mathcal{O}A \to \mathcal{O}B)$	✓	✓	✓	✓
(D)	$\mathcal{O}A \to \neg\mathcal{O}\neg A$			✓	✓
(Nec)	$A \Rightarrow \mathcal{O}A$		✓		✓
(Mono)	$A \to B \Rightarrow \mathcal{O}A \to \mathcal{O}B$	✓	✓	✓	✓

been explored by McArthur (1981), Mares (1992), Goble (2001), and Lokhorst (2006; 2008). The other way is by adding a primitive singulary operator, $\mathcal{O}$, to the language. We will focus on the latter way, the former being more explored in the context of relevant logics.

To start, we will examine some claims from Goble (1999), one of the few discussions of deontic relevant logic using the primitive operator approach. Goble presented some deontic logics over R. We will present Goble's logics and discuss his motivations before offering our own proposal.

Goble considers four logics in total, OR.1, OR.2, DR.1, and DR.2. OR.1 adds to R the following.

- $\mathcal{O}(A \to B) \to (\mathcal{O}A \to \mathcal{O}B)$ (K)
- $(\mathcal{O}A \wedge \mathcal{O}B) \to \mathcal{O}(A \wedge B)$ (Agg)
- $A \to B \Rightarrow \mathcal{O}A \to \mathcal{O}B$ (Mono)

OR.2 adds $A \Rightarrow \mathcal{O}A$ (Nec) to OR.1. The logics DR.1 and DR.2 add $\mathcal{O}\neg A \to \neg\mathcal{O}A$ (D) to OR.1 and OR.2, respectively. The deontic principles of these logics are summarized in table 1.

The OR logics are Goble's preferred logics. He says that we should reject (D) to allow for the possibility of conflicts of obligation. He says,

> [The (D) axiom] is a consistency principle, expressing the consistency of systems of norms, the propositions that are obligatory. …In combination with other standard principles of deontic logic, it implies that if A and B are incompatible states of affairs (if A entails $\neg B$) then if A is obligatory, then B is not obligatory. Thus (D) rules out the possibility of conflicts of obligations. For that very reason, D may be challenged, since it seems obvious that normative system often are inconsistent, and, in real life, a person's obligations can conflict. (Goble, 1999, 332)

Goble goes on to argue that for the proponent of classical logic, (D) is unavoidable. If one supposes that there are conflicting obligations, $\mathcal{O}A$ and $\mathcal{O}\neg A$, then one can obtain $\mathcal{O}(A \wedge \neg A)$, by (Agg) and (B9). Next, he notes that since $(A \wedge \neg A) \to B$ is classically valid, by (Mono), $\mathcal{O}(A \wedge \neg A) \to \mathcal{O}B$ is also valid. Thus, classical logic together with some simple modal principles has the result that conflicting obligations produces *deontic triviality*, in the sense that everything is obligatory. Goble says, "This makes it seem as though (D) is unavoidable, and so one must tell another story to explain away the appearance of conflicts of obligation."[10]

Goble's response is to point out that in relevant logics, $(A \wedge \neg A) \to B$ is invalid, so the preceding argument breaks down if one adopts a relevant logic. He says, "If, therefore, deontic logic were based on a relevant propositional logic, instead of classical logic, then, …it would be easy, and non-trivializing to abandon (D)."[11] There is something right about Goble's response, but there is also something wrong about it.

Goble is correct that his logics without (D) do not result in deontic triviality in the face of conflicting obligations. He is, however, not correct that rejecting (D) is required for that result, as his logics with (D) have the same feature. Every relevant logic, including R, is *paraconsistent*, meaning that it is not the case that for all B, $(A \wedge \neg A) \to B$ is valid. One can, unproblematically, have conflicting obligations without triviality. In the context of R, and weaker logics including B, (D) can be had non-trivially with conflicting obligations, $\mathcal{O}A \wedge \mathcal{O}\neg A$, without any serious issue. DR.2 and all its sublogics allow for the possibility of conflicting obligations without the threat of obligation becoming trivialized.

To be more explicit, consider a DR.2-theory whose axioms are conflicting obligations, $\mathcal{O}p$ and $\mathcal{O}\neg p$. The resulting theory is deontically non-trivial, in the sense that not every formula is obligatory. The theory will be inconsistent, since $\neg\mathcal{O}p$ will be in the theory due to (D). Nonetheless, triviality, deontic and otherwise, is averted, unlike with KD. Thus, Goble is right that using a relevant base logic one can avoid deontic triviality in the face of conflicting obligations, but he is wrong that rejection of (D) is required for that.

The (D) axiom is arguably what is distinctive about deontic logics. Some deontic logicians feel that extending a logic with (D) is what makes a logic *deontic*.[12] Defining the permissibility operator, $\mathcal{P}$, as $\neg\mathcal{O}\neg$, as is standard, results in the reading of the (D) axiom as saying that whatever is obligatory is permissible. That seems like a fine principle, and so it is reasonable to include (D) as a deontic axiom. We can hold on to the (D) while adopting a relevant base logic to avoid certain logical moves licensed by the classical material implication.

If we have adopted a relevant base logic, the (D) axiom can still be read as a kind of consistency claim. It says that if something is obligatory, then it is not impermissible. If one is using DR.1 as one's relevant deontic logic, then one can get a more explicit consistency claim by reasoning as follows. $B \vee \neg B$ is a theorem of R, so $\mathcal{O}\neg A \vee \neg\mathcal{O}\neg A$ is as well. Every standard relevant logic is closed under the rule $B \vee C$, $B \to D \Rightarrow D \vee C$, so from this rule and (D), we get $\neg\mathcal{O}A \vee \neg\mathcal{O}\neg A$ as a theorem. Using De Morgan's law, we then obtain $\neg(\mathcal{O}A \wedge \mathcal{O}\neg A)$. From (Mono), (B2), (B3), (B8), and (B9), we get $\mathcal{O}(A \wedge \neg A) \to (\mathcal{O}A \wedge \mathcal{O}\neg A)$. Whence by modus tollens, we get $\neg\mathcal{O}(A \wedge \neg A)$. With R as the base logic, (D) does yield a claim plausibly read as saying that obligations are consistent. There, however, are two points to note about this. First, just as having $\neg(A \wedge \neg A)$ as a theorem of R does not rule out non-trivial, inconsistent R-theories,

[10] (Goble, 1999, 332)

[11] (Goble, 1999, 332)

[12] This is presented as the consensus view by McNamara and Van De Putte (2025) and (Garson, 2018, §3). This is not, however, to say that no deontic logicians have contested (D).

having $\neg\mathcal{O}(A \wedge \neg A)$ in the logic does not rule out non-trivial theories containing inconsistent or conflicting obligations.

The second point is that the derivation of the consistency claim depends on logical features not shared by all relevant logics. Many standard relevant logics lack excluded middle, $A \vee \neg A$, and non-contradiction, $\neg(A \wedge \neg A)$, as theorems. For example, the logic RW, obtained by dropping $(A \to (A \to B)) \to (A \to B)$ from the given axiomatization of R, is one such logic. If we substitute RW for R in Goble's DR.1, then the derivation breaks down and $\neg\mathcal{O}(A \wedge \neg A)$ is not a theorem.[13] Our point here is that while (D) can be understood as a kind of consistency statement concerning obligation, the consequences of that understanding do not fully align with the classical understanding of (D), as is typical when working with a paraconsistent logic.

Goble takes the contrary-to-duty paradox, which he takes to supply some reason for adopting a relevant logic as one's base logic. Examining Goble's argument here will be illuminating, so we will spend some time on it. The contrary-to-duty paradox is based on four claims: (i) it ought to be that p, (ii) it ought to be that if p, then q, (iii) if not p, then it out to be that not q, and (iv) it is not the case that p. These are, intuitively, consistent and mutually independent, and they are reasonably formalized as follows.

(i) $\mathcal{O}p$
(ii) $\mathcal{O}(p \to q)$
(iii) $\neg p \to \mathcal{O}\neg q$
(iv) $\neg p$

We can then reason as follows. From (ii) and (K), we get $\mathcal{O}p \to \mathcal{O}q$. Then, with (1), we obtain $\mathcal{O}q$. But, (iii) and (iv) together yield $\mathcal{O}\neg q$. Thus, from a plausibly consistent set of claims, we reach conflicting obligations, which (D) can convert into outright inconsistency. Alternative formalizations of (ii) and (iii), $p \to \mathcal{O}q$ and $\mathcal{O}(\neg p \to q)$ respectively, appear to save consistency at the cost of violating independence. The alternative formalization of (ii), $p \to \mathcal{O}q$, follows classically from (iv), as $\neg A \to (A \to B)$ is classically valid, and the alternative formalization of (iii), $\mathcal{O}(p \to q)$, follows classically from (i), as $A \to (\neg A \to B)$ is classically valid and the rule (Mono) is available in KD.

The use of a relevant base logic appears to adequately address this paradox. The alternative formulations of (ii) and (iii) do not follow from (iv) or (i), as the required implications, namely $\neg A \to (A \to B)$ and $A \to (\neg A \to B)$, are not theorems of R. The formulations of (i)-(iv) are still inconsistent over a relevant deontic logic containing (K), but there are two points to observe here. First, as remarked above, inconsistency is not disastrous in relevant logics, as an inconsistency need not trivialize the theory. Second, the derivation of the inconsistency above used the (K) axiom. An alternative derivation is available in R. Since $(A \wedge (A \to B)) \to B$ is a theorem of R, by (Mono) and (Agg), $(\mathcal{O}A \wedge \mathcal{O}(A \to B)) \to \mathcal{O}B$ is also a theorem. Therefore, for any deontic extension L of R closed under (Mono) and containing (Agg), every L-theory containing (i) and (ii) above will also contain $\mathcal{O}q$, resulting in a contradiction when

[13] This can be verified by using John Slaney's program MaGIC, which we leave to the curious reader.

paired with (iii) and (iv).[14] Importantly, $(A \wedge (A \to B)) \to B$ is not a theorem of all relevant logics, in particular not of B. So, there are a wide range of relevant deontic logics that can take (i)–(iv) above to be both consistent and mutually independent.[15]

Goble's proposed relevant deontic logics vindicate the independence intuition in the contrary-to-duty paradox for the alternative formalizations of (ii) and (iii). If one weakens the base logic slightly, relevant deontic logics can also vindicate the consistency intuition for the above formalizations. Thus, we agree with Goble's suggestion that the contrary-to-duty paradox provides some reason for considering a relevant deontic logic, although we think that Goble did not follow this idea far enough.

To take stock, Goble presented four deontic logics, along with an argument against the (D) axiom and arguments in favor of using a relevant logic as the base logic. We agree with Goble's arguments in favor of a relevant logic, although these arguments do not neatly motivate his choice of R. We disagree with Goble's rejection of (D). On the contrary, there seem to be good reasons for including (D) in one's deontic logic.

At this point, it will be useful to have models to interpret $\mathcal{O}$. To interpret a primitive $\mathcal{O}$ operator, we will add a binary accessibility relation S to the ternary relational frames.

Definition 4.1 A *deontic ternary relational frame* is a quintuple $\langle K, N, R, S, ^* \rangle$ where $\langle K, N, R, ^* \rangle$ is a ternary relational frame and $S \subseteq K \times K$ is such that

- for all $a, b, c \in K$, if $a \leq b$ and Sbc, then Sac, and
- for all $a \in K$, there is $b \in K$ such that Sab and Sa^*b^*.

A *deontic ternary relational model* is a deontic ternary relational frame equipped with a valuation function V such that if $a \leq b$ and $a \in V(p)$, then $b \in V(p)$.

The verification condition for $\mathcal{O}$ is

- $a \Vdash \mathcal{O}B$ iff for all $b \in K$, if Sab, then $b \Vdash B$.

The logic that is validated by the class of all deontic ternary relational frames is the logic B.D, which adds to B the following axioms and rule.

- $(\mathcal{O}A \wedge \mathcal{O}B) \to \mathcal{O}(A \wedge B)$ (Agg)
- $A \to B \Rightarrow \mathcal{O}A \to \mathcal{O}B$ (Mono)
- $\mathcal{O}\neg A \to \neg\mathcal{O}A$ (D)

We get soundness and completeness, as well as variable sharing for B.D.[16]

Having looked at Goble's motivations and his proposals, we offer a counterproposal for minimal deontic relevant logics. Let L be a relevant logic, which for our purposes can be any sublogic of R. The *basic deontic logic* over L, L.D, is obtained by adding the principles principles above, (Agg), (Mono), and (D), to L. Where L is sound and complete with respect to a class of ternary relational frames, L.D is sound and complete with respect to the corresponding class of deontic ternary relational frames. One can

[14] We will note that all standard relevant modal logics are closed under (Mono) and contain (Agg). See Fuhrmann (1990) for more.

[15] This remains true if one strengthens the base logic a lot from B. For example, one can strengthen the base logic to the logic RW, mentioned earlier in this section.

[16] Fuhrmann (1990) proves soundness and completeness. We will return to variable sharing below

strengthen L.D by adding additional axioms or rules. We will focus on (K) and (Nec), since those are discussed by Goble. The addition of either, or both, of (K) and (Nec) to L.D results in a logic that enjoys variable sharing. Soundness and completeness for L.D with respect to an appropriate class of deontic ternary relational frames is maintained for the extension with (K) or (Nec), provided one imposes corresponding frame conditions for the deontic principles.

Of Goble's basic principles for OR.1, (Agg) and (Mono) hold for any logic defined by a class of ternary relational frames equipped with a binary relation S. Crucially, (K) is not in general valid. On the contrary, its validity requires a substantive frame condition, to which we return below. Goble (1999) does not provide any compelling reason for accepting (K). In fact, analysis of the contrary-to-duty paradox reveals that there is a good reason for rejecting (K), namely that rejecting it allows one to maintain the consistency of the starting principles, provided the base logic is sufficiently weak.[17] The (K) axiom need not be included in anything claiming to be a plausibly minimal relevant deontic logic.

One of the two axes along which Goble distinguishes logics is whether they are closed under the rule (Nec). (Nec) is not proposed for the minimal logics, OR.1 and DR.1. We agree with Goble that (Nec) should not be included in the minimal logic. As with (K), (Nec) requires a substantive frame condition for its validity in ternary relational frames.

Avoiding (Nec) seems good. Prior to the advent of the use of Kripke frames, deontic logic was studied primarily via axiom systems, and many deontic logicians are opposed to (Nec) as a rule for an obligation operator.[18] When logics are defined by classes of Kripke frames, (Nec) is built into the logic by the framework, so it must be accepted, even if it is not a virtue. In the context of deontic ternary relational frames, however, we can reject it, so that seems good.

We note that the condition for the (D) axiom is

- For all $a \in K$, there is $b \in K$ such that Sab and Sa^*b^*,

rather than the regular seriality condition, which omits the second conjunct. Regular seriality is not sufficient for the validity of (D) in the setting of ternary relational frames with a binary modal accessibility relation, and, in fact, can be added to the basic frames without a resulting change to the logic.[19]

The conditions for (K) and for (Nec) are, respectively,

- For all $a, b, c \in K$, if $\exists z(Rabz$ and $Szc)$, then $\exists x\exists y(Sax$ and Sby and $Rxyc)$, and
- for all $a, b \in K$, if Sab and $a \in N$, then $b \in N$.

One gets soundness and completeness with respect to appropriate classes of frames obeying these conditions as well.

[17] We would like to thank an anonymous referee for raising this issue.

[18] See for example, von Wright (1951). See also the discussion by Humberstone (2016, 235), especially remark 4.4.9, and by Garson (2013, 46). Belnap et al. (2001, 303) restrict the formulas to which $\mathcal{O}$ can be attached, and they remark that (Nec) holds for $\mathcal{O}$ only because none of those formulas are logically valid, saying, "Perhaps it will come as a relief to learn that we are under no obligation to see to it that 2+2=4."

[19] Mares (1994) proves this latter claim.

While we agree with some of Goble's motivations for adopting a relevant deontic logic, we do not think that his proposals are well motivated. Instead of the systems OR.1 and DR.1, we have argued that L.D presents a better candidate for the minimal deontic extension of a relevant logic L. In the case of B, the minimal deontic logic is B.D, and for R, it is R.D. R.D differs from Goble's DR.1 and OR.1 in not having the (K) axiom built in. It further differs from DR.2 and OR.2 in its lack of (Nec).

Having isolated a plausible candidate for the minimal deontic logic over L, let us turn to a discussion of the principle that ought implies can.

5 Possibility

A natural thought when working in deontic logic is to add a possibility operator $\Diamond$ to the language. This will allow one to express the thesis that ought implies can,

- $\mathcal{O}A \to \Diamond A$,

which is called "Kant's law" by Routley and Plumwood (1989).[20]

Routley and Plumwood think that the idea that the obligations in deontic logics should be consistent is "*fundamentally* mistaken" and should be rejected.[21] They say,

> This Kantian assumption [$\mathcal{O}A \to \Diamond A$] should be rejected: moral dilemmas are part of the data and no illusion, and no amount of reflection need budge them. However the Kantian ought-implies-can theme makes it difficult to retain moral dilemmas as part of the data. For it immediately converts moral dilemmas, which induce no inconsistency, into explicit contradiction. …It is this immediate conversion of dilemmas to explicit inconsistency that lies behind the widespread idea that moral dilemmas themselves reflect 'some kind of inconsistency'.[22]

It is worth unpacking the ideas of Routley and Plumwood to see what they have right and what they have wrong. To start, (D) converts conflicting obligations into contradictions. Since we have $(\mathcal{O}A \wedge \mathcal{O}\neg A) \to \mathcal{O}\neg A$ from B, with $\mathcal{O}\neg A \to \neg\mathcal{O}A$ from (D) and some transitivity moves available in B, we obtain $(\mathcal{O}A \wedge \mathcal{O}\neg A) \to (\mathcal{O}A \wedge \neg\mathcal{O}A)$. So, one need not even invoke Kant's law to turn conflicting obligations into explicit contradictions. Thus, the criticism here of Kant's law here by Routley and Plumwood is misplaced.

If we set aside (D), an alternative route to contradiction from conflicting obligations is the following. From $\mathcal{O}A \wedge \mathcal{O}\neg A$ we can get $\mathcal{O}(A \wedge \neg A)$ by (Agg). An instance of Kant's law is $\mathcal{O}(A \wedge \neg A) \to \Diamond(A \wedge \neg A)$. These yield an explicit contradiction if we add the plausible claim that contradictions are impossible $\neg\Diamond(A \wedge \neg A)$. This last is an additional, unstated assumption, and it does not come for free in the frames without further conditions, a point to which we will return below. So, one can get to a contradiction from conflicting obligations via Kant's law. The question for Routley and Plumwood is what the significance of that fact is.

[20] For some further discussion of Kant's law, see Russell (2023, 139ff.) and Weiss (2025).

[21] Routley and Plumwood (1989, 658), emphasis in the original.

[22] Routley and Plumwood (1989, 6745)

If one is working with classical logic, then contradictions, whether arising from obligations or the impossible, imply everything. Any classical theory containing a contradiction will be trivial, in the sense of containing every formula. If one is working with a relevant logic, or generally a paraconsistent logic, contradictions need not imply everything. Even when one has consistency principles, such as $\neg\Diamond(A \wedge \neg A)$ and $\neg\mathcal{O}(A\wedge\neg A)$, in one's logic, contradictions need not imply everything. Contradictions, together with the consistency principles, can lead to more contradictions, but this need not trouble the relevant logician. As we have noted, one can have inconsistent but non-trivial theories in a relevant logic.

To illustrate, suppose that L is a suitable relevant deontic logic and one has an L-theory in which p is impossible but obligatory, i.e. $\neg\Diamond p \wedge \mathcal{O}p$. Given contraposition, it follows from Kant's law, p is not obligatory, $\neg\mathcal{O}p$. But, the initial obligation, $\mathcal{O}p$, does not disappear from the theory. Instead, we reach contradictory obligations, namely $\mathcal{O}p$ and $\neg\mathcal{O}p$, but the agent is not thereby exonerated from her initial obligation, $\mathcal{O}p$.

Routley and Plumwood seem to make a similar mistake one Goble does.[23] The fact that the logic has some principles that result in a contradiction with conflicting obligations is a problem for classical logic. It need not, however, be a problem for paraconsistent and relevant logics. A contradiction certainly need not trivialize in the context of relevant deontic logics. Kant's law may result in the logic saying that conflicting obligations are contradictory and impossible. They may also have models in which they are true and are also possible (as well as impossible). There is no reason that a paraconsistent logic has to shy away from contradictions here.

There is a question about how Routley and Plumwood are thinking of interpreting the possibility operator. One verification condition that they might use is the universal one:

- $a \Vdash \Diamond B$ iff for some $b \in K$, $b \Vdash B$.

This interpretation of possibility does lead to problems, as contradictions involving $\Diamond$ explode: $(\Diamond A \wedge \neg\Diamond A) \to B$ becomes valid. This follows from the discussion of universal necessity by Standefer (2022; 2023), but we will present a short proof here.[24] Suppose that for some model, $a \Vdash \Diamond A \wedge \neg\Diamond A$. Then $a \Vdash \Diamond A$ and $a \Vdash \neg\Diamond A$, so $a^* \nVdash \Diamond A$. This implies that for all $b \in K$, $b \nVdash A$, but $a \Vdash \Diamond A$ implies there is $c \in K$ such that $c \Vdash A$, which is a contradiction. Therefore, there is no counterexample to $(\Diamond A \wedge \neg\Diamond A) \to B$.

Under the universal interpretation, a contradiction involving possibility would be a problem. Given Kant's law, a theory with $\mathcal{O}p$ and $\mathcal{O}\neg p$ would also contain $\Diamond(p \wedge \neg p)$. If the theory also contains the claim that contradictions are impossible, $\neg\Diamond(p \wedge \neg p)$, then the universal interpretation of possibility leads to triviality, as $(\Diamond(p \wedge \neg p) \wedge \neg\Diamond(p \wedge \neg p)) \to B$, for all formulas B is valid. It is worth noting, however, that the underlying issue is independent of Kant's law, namely the fact that contradictions involving possibility imply everything. The logic does not imply that there are conflicting obligations, as that is something that does not happen even with

[23] It is worth noting that while Routley and Plumwood focus their discussion on classical logic for part of their article, the quotations above come from a discussion focused on relevant logics.

[24] See also the further investigation of universal necessity and possibility by Standefer and French (2025).

taking classical logic as the base logic. The issue, we think, is that the relevant logician should not be using the universal interpretation of possibility.

Rather than adopt the universal interpretation of possibility, the relevant logician can (and should) adopt a different interpretation. Let us say that a *modal deontic frame* is a deontic ternary relational frame equipped with a binary relation T on K subject to the condition

- For all $a, b, c \in K$, if $a \leq b$ and Tac, then Tbc.

Given this, one can use the following verification condition for possibility.

- $a \Vdash \Diamond B$ iff for some $b \in K$, Tab and $b \Vdash B$.

With this verification condition, $(\Diamond A \wedge \neg\Diamond A) \rightarrow B$ will be invalid. There will, therefore, be nothing requiring conflicting obligations to lead to triviality. Conflicting obligations will, in no sense, be rendered illusory.

While the universal interpretation of the possibility operator does result in Kant's law being valid, the interpretation using T does not without an additional condition, $T \subseteq S$, which can be adopted or not. Further, $\neg\Diamond(A \wedge \neg A)$ is not generally valid either. To clarify, we are not suggesting that these principles should be rejected, but rather we are highlighting what is built into the framework and what is not, which can suggest directions for further investigation.

Kant's law should not be the focus of Routley and Plumwood's objections, at least not for the reasons given above. They complain that conflicting obligations, or moral dilemmas, $\mathcal{O}A \wedge \mathcal{O}\neg A$, are converted into explicit contradictions. As noted above, this conversion can be done using just (D) together with some basic features of B. If we look at the route that goes via Kant's law, we see that we used $\neg\Diamond(A \wedge \neg A)$ and (Agg). If their concern is with the conversion conflicting obligations into obligatory contradictions, i.e. from $\mathcal{O}A \wedge \mathcal{O}\neg A$ to $\mathcal{O}(A \wedge \neg A)$, then they could reasonably reject (Agg). This move would require adopting different frames for the deontic logic, such as the use of neighborhood functions, rather than accessibility relations.[25] This is to say that one can be moved by the objection of Routley and Plumwood, that moral dilemmas need not involve explicit contradictions, while also holding onto Kant's law in the setting of relevant deontic logic. Indeed, this is a move open to the classical deontic logician as well.[26]

We conclude that the relevant logician can include Kant's law in their deontic logic, but care is needed concerning the sort of possibility operator at issue. Let us now turn to some ways of strengthening the relevant deontic logic L.D.

6 Strengthening the Logic

In §4, we argued in favor of a relevant analog of KD, which is singled out as a distinguished weakest deontic logic in the classical setting. For a relevant logic L, its minimal deontic strengthening is L.D. In this section, we turn to the other end of the

[25] For more on neighborhood frames and relevant logics, see Standefer (2019), Tedder and Ferenz (2022), Ferenz (2023), and Ferenz and Tedder (2023; 2025).

[26] See, for example, Goble's (2000) multiplex models for (classically based) deontic logic.

spectrum and consider strong logics. In the setting of classically based deontic logic, there is a logic that is singled out as a natural strong logic. That logic is KD45 which adds to KD the axioms

- $\mathcal{O}A \to \mathcal{O}\mathcal{O}A$ (4) and
- $\neg\mathcal{O}A \to \mathcal{O}\neg\mathcal{O}A$ (5).

As noted in §2, the Kripke frames for this logic obey the conditions of seriality, transitivity, and Euclideanness. We can describe this in a different, suggestive way if we define some notation. Let $S(w) = \{u \in W : Swu\}$. Given that, in the Kripke frames for this logic, each world w is assigned a non-empty set $S(w) \subseteq W$ where S is universal over $S(w)$, i.e. for $x, y \in S(w)$, $S(x) = S(y)$. The intuitive idea here is that each world has a set of normatively ideal worlds, and being obligatory is holding in all of those. The logic KD45 is sound and complete with respect to these frames.

The logic KD45 is also sound and complete with respect to semi-simplified frames $\langle W, X\rangle$, where $\emptyset \neq X \subseteq W$.[27] In these frames, we use the verification condition

- $w \Vdash \mathcal{O}A$ iff for all $x \in X$, $x \Vdash A$.

The idea is that the elements of X are the normatively ideal worlds of the frame, and being obligatory is being true in all those worlds. In the semi-simplified frames, each frame has a single set of ideal worlds, whereas in the general Kripke frames, different bunches of worlds may be normatively ideal from the point of view of different worlds. We will explore these two approaches to deontic logics below.

To start, let us consider semi-simplified ternary relational frames.

Definition 6.1 A semi-simplified ternary relational frame $\langle K, N, R, X, ^*\rangle$ is a ternary relational frame equipped with a non-empty subset $X \subseteq K$.

A semi-simplified ternary relational model adds a valuation V to a semi-simplified ternary relational frame F such that if $a \leq b$ and $a \in V(p)$, then $b \in V(p)$.

Following on the idea above, we use the following verification condition for $\mathcal{O}$,

- $a \Vdash \mathcal{O}A$ iff for all $b \in X$, $b \Vdash A$.

The definitions of counterexample and validity are adapted in a straightforward way.

Using the above condition, we get a violation of variable sharing. This is easy to see using the following lemma.

Lemma 6.1 *Let M be a semi-simplified ternary relational model. If $a \Vdash \mathcal{O}A$, then for all $c \in K$, $c \Vdash \mathcal{O}A$*

Proof Suppose $a \Vdash \mathcal{O}A$. Then for all $b \in X$, $b \Vdash A$. Let $c \in K$ be arbitrary. It then follows that $c \Vdash \mathcal{O}A$. □

This means that in a model, the set of worlds where $\mathcal{O}A$ holds is either K or $\emptyset$. We get the following corollary straight away.

Corollary 6.1 *$(\mathcal{O}A \wedge \neg\mathcal{O}A) \to B$ is valid in the class of all semi-simplified ternary relational frames.*

[27] See Pietruszczak (2009) and Pietruszczak et al. (2020), for example.

From this, we get the validity of $(\mathcal{O}p \wedge \neg\mathcal{O}p) \rightarrow q$.[28] We conclude that the relevant logician should not adopt semi-simplified ternary relational frames for deontic logic. Rather than continue to explore the logic of semi-simplified ternary relational frames, we will turn to other options for properly relevant D45-type deontic logics.

Given a relevant logic L, we have two options to strengthen the logic to something analogous to a D45-type extension. One is to take the axioms (D), (4), and (5) and impose appropriate frame conditions on the deontic frames from §4. The other is to take the deontic frames ternary relational frames and impose transitivity and Euclideanness on them. As we will see, these two options lead to two different logics in the present setting.

Let us start with the first option. To get frames for L.D extended by (4) and (5), we need to restrict to deontic ternary relational frames in which S obeys the following extra conditions.

(F4) For all $a, b, c \in K$, if Sab and Sbc, then Sac.
(F5) For all $a, b, c \in K$, if Sa^*c and Sab, then Sb^*c.

We can add to L.D the (4) and (5) axioms. Let us call the resulting logic L.D45. As shown by Furhmann (1990), we have soundness and completeness for L.D45 with respect to the class of deontic frames for L.D obeying the conditions above.

Where L is a relevant logic that is a sublogic of R, L.D45 enjoys the variable-sharing property. To show this, we will appeal to a result proved by Standefer (2023). That result concerns the logic RS5, which is a proper extension of R.D45, and so contains L.D45, for any relevant logic L contained in R.[29]

Theorem 1 *The logic* RS5 *enjoys the variable-sharing property.*

Corollary 6.2 *For any sublogic* L *of* R*,* L.D45 *enjoys the variable-sharing property.*

So, for any sublogic L of R, L.D45 is, plausibly, a relevant deontic logic. R.D45, and many other choices for L, are sound and complete with respect to a class of deontic ternary relational frames. Note, however, that the frames in question obey a condition, (F5), that is not quite the Euclidean condition associated with (5) in classical Kripke frames. Instead, this condition has the Routley star in two places. This difference makes an important difference, as we will see shortly.

Let us now turn to the second option for D45-type relevant deontic logics. For this, we can adopt something closer to the semi-simplified frames, although modified to cohere with the underlying relevant logics better, or in other words, that respects variable sharing. Rather than a single designated set of ideal worlds, we allow each world to have its own set of ideal worlds, as with the Kripke frames for classically based KD45.

Definition 6.2 An ideal worlds ternary relational frame $\langle K, N, R, S, ^* \rangle$ is a ternary relational frame equipped with a binary relation $S \subseteq K \times K$ such that

(S1) For all $a, b, c \in K$, if $a \leq b$ and Sbc, then for some $x \in K$, $x \leq c$ and Sax.

[28] We can also get as a corollary that $q \rightarrow (\mathcal{O}p \rightarrow \mathcal{O}p)$ is valid, so negation is not essential to the issue.

[29] In particular, RS5 adds to R.D45 the principles (K), (Nec), and (T) ($\mathcal{O}A \rightarrow A$). Note that RS5 is not intended as a deontic logic.

(S2) For all $a, b \in K$, if Sab and Sbc, then Sac.
(S3) For all $a, b \in K$, if Sab and Sac, then Sbc.
(S4) For all $a \in K$, there is $b \in K$, Sab and Sa^*b^*.

An ideal worlds ternary relational model adds a valuation V to an ideal worlds ternary relational frame F such that if $a \leq b$ and $a \in V(p)$, then $b \in V(p)$.

The verification condition we use is then the following.

- $a \Vdash \mathcal{O}A$ iff for all $b \in K$, if Sab, $b \Vdash A$

Let the logic of the class of all ideal worlds ternary relational frames be called B.IW.

In the ideal worlds frame, each $b \in K$ is assigned a non-empty set, $S(b) \subseteq K$. For all $c, d \in S(b)$, Scd, so S is universal on the set $S(b)$. To see this, note that $c \in S(b)$ and $d \in S(b)$ mean, respectively, Sbc and Sbd. By the Euclidean condition, (S3), Scd, as desired. So, each $b \in K$ is assigned a set of ideal worlds, although there can be different sets assigned to different points in the frame.[30]

We now verify that B.IW contains some principles.[31]

Theorem 2 *The following are valid in the class of ideal worlds ternary relational frames.*

(1) $\mathcal{O}\neg A \to \neg\mathcal{O}A$
(2) $\mathcal{O}A \to \mathcal{O}\mathcal{O}A$
(3) $\mathcal{O}(A \vee \mathcal{O}B) \to (\mathcal{O}A \vee \mathcal{O}B)$

Proof For (1), suppose that $a \Vdash \mathcal{O}\neg A$. Then, for all $d \in K$, if Sad, $d \Vdash \neg A$. Suppose that $a \nVdash \neg\mathcal{O}A$. Then $a^* \Vdash \mathcal{O}A$, so for all $c \in K$, Sa^*c, $c \Vdash A$. From condition (S4), there is $b \in K$, such that Sab and Sa^*b^*. From Sab, it follows that $b \Vdash \neg A$ so $b^* \nVdash A$. From Sa^*b^*, it follows that $b^* \Vdash A$, which is a contradiction.

For (2), suppose $a \Vdash \mathcal{O}A$. It follows that for all $b \in K$, if Sab, $b \Vdash A$. Suppose Sbc. By condition (S2), Sac, so $c \Vdash A$. This implies $b \Vdash (\mathcal{O}A)$, from which we conclude $a \Vdash \mathcal{O}\mathcal{O}A$.

For (3), suppose $a \Vdash \mathcal{O}(A \vee \mathcal{O}B)$. Suppose $a \nVdash \mathcal{O}A \vee \mathcal{O}B$. This implies that there are $b, c \in K$, such that Sab, Sac, $b \nVdash A$, and $c \nVdash B$. Since Sab, $b \Vdash A \vee \mathcal{O}B$, whence $b \Vdash \mathcal{O}B$. By condition (S3), Sbc, so $c \Vdash B$, which is a contradiction. □

It will be helpful to present a frame that allows us to demonstrate an invalidity. The frame F_1, adadpted from Standefer (2026a), is presented in table 2. In this frame, $K = \{0, a, b\}$ and $N = \{0\}$. The remaining portions of the frame are given in the

[30] We note that there is something slightly inaccurate in calling $S(b)$ a set of "ideal worlds", as the points in $S(b)$ are, generally, not well thought of as worlds. The points of the models can be incomplete and inconsistent, and so they are not quite world-like. There is a tradition in relevant modal logic of including a set of special points in the frame that are world-like in the sense of being complete and consistent. See, for example, Sedlár (2015), Sedlár and Vigiani (2022; 2024), or Ferenz (2024).

[31] The final principle (OM), $\mathcal{O}(A \vee \mathcal{O}B) \to (\mathcal{O}A \vee \mathcal{O}B)$, has been studied by Ono (1977) in the context of S5-type extensions of intuitionistic logic. The name (OM) is not standard in the literature. Indeed, there is no standard name for this axiom in classically based modal logics, as it is interderivable with (5), given the principles of K. The name stands for 'Ohnishi Matsumoto', because it is closely connected to the right rule for $\Box$ in the S5 sequent system of Ohnishi and Matsumoto (1957).

Table 2 Frame F_1

R	0	a	b	*	S
0	0	a	b	0	$0b$
a	a	a	$0ab$	b	a
b	b	$0ab$	b	a	$0b$

table. To explain the R portion of the table, the row headers indicate the first position in the ternary relation, the column headers the second position, and the cells in the middle indicate all the points that occupy the third position. So, the b-row, a-column entry '$0ab$', means that $Rba0$, $Rbaa$, and $Rbab$ hold. Similarly, '$0b$' under the 0-row for S means $S00$ and $S0b$. This is an ideal worlds ternary relational frame.[32]

Lemma 6.2 (5) *is invalid for ideal worlds ternary relational frames.*

Proof Take the model on F_1 where $V(p) = \{b, 0\}$. Then, $a \nVdash \mathcal{O}p$, so $b \Vdash \neg\mathcal{O}p$. Since $0 \Vdash \mathcal{O}p$, $0 \nVdash \neg\mathcal{O}p$. It then follows that $b \nVdash \mathcal{O}\neg\mathcal{O}p$. This suffices for $0 \nVdash \neg\mathcal{O}p \to \mathcal{O}\neg\mathcal{O}p$. □

This lemma has consequences for the relationship between B.IW and B.D45. (5) is not a theorem of B.IW, as shown by the lemma, although it is a theorem of B.D45. The axiom (OM), $\mathcal{O}(A \vee \mathcal{O}B) \to (\mathcal{O}A \vee \mathcal{O}B)$, while a theorem of B.IW, is not a theorem of B.D45.[33] This means that two plausible candidates for strengthening the basic relevant deontic logic are incomparable.

What about the question of axiomatizing B.IW? Demonstrating completeness for a proposed axiom system is difficult as the usual Henkin-style completeness proofs do not seem to work. An adaptation of these methods, used by Standefer and Mares (2025) to prove completeness for an axiomatization of the equivalence relation logic of Standefer (2026a), does not appear to carry over to the ideal worlds ternary relational frames. We will, then, leave the question of axiomatizing B.IW for future work.

We can, however, get a completeness result for a subclass of frames. Following Standefer (2026a), let us define coordinated ideal world frames.

Definition 6.3 A coordinated ideal worlds frames is an ideal worlds ternary relational frame obeying the additional condition

(S5) For all $a, b \in K$, if Sab, then Sa^*b^*.

Let us call the logic of the coordinated ideal worlds frames B.CIW. Next, let us define Mares's (1994) mostly Meyer modal frames.[34]

[32] It is actually a frame for the logic R, but that is more than we need here.

[33] This fact can be shown using a MaGIC countermodel, which we will omit here.

[34] The original mostly Meyer modal frames from Mares (1994) are for the logic RK^-. This is a logic that extends R with, in our nomenclature, (Agg), (Mono), (MC), and (Nec). The frames presented here are generalizations of Mares's in two ways. First, they are not restricted to ternary relational frames satisfying the frame conditions for R. Second, Mares's frames satisfy an additional frame condition for (Nec), which he calls $R\Box 2$. In our notation, this would be if $a \in N$ and $b \in S(a)$, then $b \in N$. This is an optional condition that can be dropped from Mares's frames, and soundness and completeness can be maintained by dropping the (Nec) rule from the axiomatizations.

Definition 6.4 A mostly Meyer modal frame is a quintuple $\langle K, N, R, S, ^* \rangle$ where $\langle K, N, R, ^* \rangle$ is a ternary relational frame and $S \subseteq K \times K$ is such that

(MM1) for all $a, b, c \in K$, if $a \leq b$ and Sbc, then there is $x \in K$ such that Sax and $x \leq c$, and
(MM2) for all $a, b \in K$, if Sab, then Sa^*b^*.

We can see that coordinated ideal worlds frames are mostly Meyer modal frames subject some additional conditions.

Lemma 6.3 *All mostly Meyer modal frames subject to the additional conditions*

(MM3) for all $a \in K$, there is $b \in K$ such that Sab,
(MM4) for all $a, b, c \in K$, if Sab and Sbc, then Sac, and
(MM5) for all $a, b, c \in K$, if Sab and Sac, then Sbc

are coordinated ideal worlds frames.

Proof The only condition that needs additional verification is (S4). From (1), Sab, so by condition (MM2), Sa^*b^*, so the frame satisfies condition (S4). □

So, the coordinated ideal worlds frames and the mostly Meyer modal frames subject to conditions (MM3)–(MM5) are the same class of frames. This means that we can leverage the completeness proof of Mares (1994) to obtain a completeness result. First, we need to provide the axiom system.

Let HB.CIW be the logic axiomatized by the adding the following axioms and rule to B.

- (Agg) $(\mathcal{O}A \wedge \mathcal{O}B) \rightarrow \mathcal{O}(A \wedge B)$
- (D) $\mathcal{O}A \rightarrow \neg\mathcal{O}\neg A$
- (4) $\mathcal{O}A \rightarrow \mathcal{O}\mathcal{O}A$
- (5) $\neg\mathcal{O}A \rightarrow \mathcal{O}\neg\mathcal{O}A$
- (MC) $\mathcal{O}(A \vee B) \rightarrow (\mathcal{O}A \vee \neg\mathcal{O}\neg B)$
- (Mono) $A \rightarrow B \Rightarrow \mathcal{O}A \rightarrow \mathcal{O}B$

Now, we can claim completeness.

Theorem 3 *The logic* HB.CIW *is sound and complete with respect to mostly Meyer modal frames obeying the conditions*

(MM3) for all $a \in K$, there is $b \in K$ such that Sab,
(MM4) for all $a, b, c \in K$, if Sab and Sbc, then Sac, and
(MM5) for all $a, b, c \in K$, if Sab and Sac, then Sbc.

Proof The proof follows that of Mares (1994). The details are basically the same, so we omit them. □

Corollary 6.3 HB.CIW=B.CIW.

While we have not axiomatized B.IW, we have axiomatized a stronger logic, B.CIW.[35] B.CIW extends both B.IW and B.D45. It is, we suggest, a natural, stronger relevant deontic logic. The distinctive principle of B.CIW is (MC), $\mathcal{O}(A \vee B) \rightarrow (\mathcal{O}A \vee \neg\mathcal{O}\neg B)$.

[35] The reader may be wondering what happened to the axiom (OM) $\mathcal{O}(A \vee \mathcal{O}B) \rightarrow (\mathcal{O}A \vee \mathcal{O}B)$ in HB.CIW. It follows from (MC) and (5), together with some background facts about B.

Table 3 Deontic principles of the logics

Name	Modal principle	B.D	B.D45	B.IW	B.CIW
(Agg)	$(\mathcal{O}A \wedge \mathcal{O}B) \rightarrow \mathcal{O}(A \wedge B)$	✓	✓	✓	✓
(D)	$\mathcal{O}A \rightarrow \neg\mathcal{O}\neg A$	✓	✓	✓	✓
(4)	$\mathcal{O}A \rightarrow \mathcal{O}\mathcal{O}A$		✓	✓	✓
(5)	$\neg\mathcal{O}A \rightarrow \mathcal{O}\neg\mathcal{O}A$		✓		✓
(MC)	$\mathcal{O}(A \vee B) \rightarrow (\mathcal{O}A \vee \neg\mathcal{O}\neg B)$				✓
(OM)	$\mathcal{O}(\mathcal{O}A \vee B) \rightarrow (\mathcal{O}A \vee \mathcal{O}B)$			✓	✓
(Mono)	$A \rightarrow B \Rightarrow \mathcal{O}A \rightarrow \mathcal{O}B$	✓	✓	✓	✓

It says that if a disjunction is obligatory, then the disjuncts are either obligatory or permissible. This is a plausible axiom, and the resulting logic is more similar to the classically-based KD45.[36] In particular, the resulting logic of obligation and permission exhibits more of the standard duality that one sees in classically-based deontic logics, and it plays an important role in some conservative extension results.[37] One might, nonetheless, question the importance of (MC), and it is worth noting that the logic resulting from imposing (F5) on the ideal worlds frames would validate (5), but not (MC). This logic would also extend both B.D45 and B.IW, so it could provide a worthwhile point of comparison with B.CIW. Details of that comparison with B.CIW will, however, need to wait on completeness results we do not yet have.

We will summarize the principles of the different logics in table 3. B.D is a sublogic of all the other logics in the table and B.CIW has all the other logics listed as a sublogic. As noted above, B.D45 and B.IW are incomparable.

None of the logics in Table 3 have (K) or (Nec). In §4, we noted that B.D can be extended with (K) or (Nec). The same point applies here, and the same soundness and completeness points apply. We think there are sound philosophical reasons for not adding either of those principles, but we note the option is there.

We will end this section with a variable-sharing result. Let us say that R.CIW is obtained by extending B.CIW with the R axioms.

Theorem 4 R.CIW *enjoys the variable-sharing property.*

Proof The proof of theorem 4 from Standefer (2020) shows that R.CIW enjoys the variable-sharing property. □

As a corollary, all the logics summarized in table 3 enjoy the variable-sharing property. This is maintained even with the addition of (K) and (Nec). It is also maintained when strengthening the base logic up to R.

[36] Ferenz (2023) discusses (MC) and its relation to classicality in the context of quantified modal relevant logics.

[37] On the first point, see Mares (1993) and his discussion of generic truth conditions. On the second point, see the results of Mares and Meyer (1992) and Mares and Tanaka (2010), for example.

7 Conclusion

In this paper, we have examined the reasons offered by Goble (1999) for his proposals for deontic relevant logics. After criticizing his proposals, we offered a suggestion for a minimal deontic relevant logic over a relevant logic L, L.D. We then discussed criticisms of Kant's law by Routley and Plumwood, arguing that their objections missed the mark. Finally, we looked at natural ways of extending L.D. In doing so, we argued that one natural way of modeling $\mathcal{O}$ does not support the variable-sharing property. We identified two logics, B.D45 and B.IW, that are incomparable, along with a logic, B.CIW, that extends both. All of these proposals omit some of the principles of Goble's systems, namely (K) and (Nec). We think that these are promising systems for use in studying normative systems. A natural next step is adding a conditional obligation operator, but we will leave that for future work.

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Declarations

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